

A Study on the Universality and Linearity of the Leavitt Law in the LMC and SMC Galaxies

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ABSTRACT

The universality and linearity of the Leavitt law are hypotheses commonly adopted in studies of galaxy distances using Cepheid variables as standard candles. In order to test these hypotheses, we obtain slopes of the Leavitt law using linear regressions of fundamental-mode Cepheids observed by the Optical Gravitational Lensing Experiment project in the Magellanic Clouds. We find that the slopes in VI -bands and in the Wesenheit index behave exponentially, indicating non-linearity. We also find that the slopes obtained using long-period Cepheids can be considered as universal in the VI -bands, but not in the Wesenheit index.

Key words: Stars: Variables: Cepheids, Magellanic Clouds, Methods: Statistical

1 INTRODUCTION

During the first decade of the twentieth century, observations of variable stars from the Small Magellanic Cloud (SMC) galaxy taken with the 24-inch Bruce telescope at Harvard Boyden Station in Arequipa, Peru, conducted to Henrietta Leavitt to make an important discovery: the correlation between the logarithm of the pulsation periods of Cepheid variables and their magnitudes (Leavitt & Pickering 1912). In honour of its discoverer, this correlation, commonly called Period-Luminosity (PL) relation, has been renamed recently as the Leavitt law (LL) (Freedman & Madore 2010). Originally, the size of the sample used by Leavitt to establish the PL relation was statistically small: only 25 Cepheids. Thirteen years later, the number of SMC Cepheids used to establish the PL relation increased by more than a factor of four (Shapley, Yamamoto & Wilson 1925). More recently, microlensing projects, as the Massive Compact Halo Objects (MACHO) and the Optical Gravitational Lensing Experiment (OGLE), discovered hundreds of Cepheids, as a result of monitoring systematically the bars of the Magellanic Clouds. Based on these observations, it was possible to determine, with high precision, periods and mean VI magnitudes of Cepheids, and the slope and zero-point values of the LL for the Large Magellanic Cloud (LMC) galaxy (Udalski et al. 1999a). These advances confirmed the LL as an useful modern tool to determine extragalactic distances, tied to an assumed LMC distance modulus of 18.50 mag (Freedman et al. 2001), as shown by several

studies in the last years, but in particular through the work of the Araucaria Project by measuring distances to selected galaxies of the Local Group and the Sculptor Group (Pietrzyński et al. 2004, Pietrzyński et al. 2010, and references therein).

In order to measure distances at galaxies through the LL, two important hypotheses have to be adopted: its universality and linearity. Universality means that the slope of the LL is independent (or weakly dependent) on metallicity, implying that this value in a photometric band is the same for all galaxies. Linearity means that there is a linear correlation between mean magnitudes and the logarithm of the pulsation periods of Cepheids.

The influence of metallicity on the PL relation has been discussed by several groups. Studying some selected galaxies, Udalski et al. (2001) and Pietrzyński et al. (2004) found strong evidence in favour of the universal slope of the PL relation in optical bands, between the metallicity range of -1.0 dex (IC1613) up to -0.3 dex (LMC). Pietrzyński et al. (2004) and Bono et al. (2010) showed that the use of the Wesenheit index, $W_I = I - 1.55(V - I)$, minimises not only the effects of reddening but also those of metallicity on the PL relations. Gieren et al. (2005) and Fouqué et al. (2007) showed that the slopes of the PL relations in VI -bands and in the W_I index do not change significantly in the Milky Way and LMC galaxies.

Although the universality and linearity of the PL relation have been assumed in distance determination studies, there are works showing that these hypotheses are not always truthful. With respect to the non-universality, Storm et al. (2011), using the infrared surface brightness technique, found that the slopes and zero points of the PL relations of

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the LMC and Milky Way present a significant dependence on metallicity in the Wesenheit index, a weak dependence on metallicity in the optical *VI*-bands, and do not present any dependence on metallicity in *K*-band.

With respect to non-linearity, the Russian astronomer Boris Kukarkin found in the thirties that the PL relations of external galaxies present a break around 10 days, as it was mentioned by Fernie (1969) and references therein. Tammann, Sandage & Reindl (2003) found a break of the Period-Colour (PC) relation at 10 days in the LMC galaxy. Kanbur & Ngeow (2004) confirmed these break based on the statistical *F*-test.

From a study of the LMC Cepheids, Kanbur et al. (2007) detected non-linearity of the LL. This was found when using the testimator method, that determines if there is a slope change in a *n*th subset with respect to a smoothed slope obtained in all previous subsets. The smoothing process is applied in order to avoid the outliers influence on the slopes computed. Applying this method, those authors detected a significant change in the slope between subsets of period around 10 days.

Koen, Kanbur & Ngeow (2007) presented a study of non-linearity of the PL relation and the Period-Luminosity-Colour (PLC) relation using two non-parametric methods: linear regression residuals and additive models. These authors found again that the PL relation is non-linear and presents a break around 10 days. Possible causes for the non-linearity of the LL were suggested by Kanbur, Ngeow & Buchler (2004) and Kanbur & Ngeow (2006).

Ngeow et al. (2012) studied the PL relation at individual phases of the pulsation cycle of Cepheids (the multi-phase approach) and found again a break around 10 days. They showed that the behaviour of the LL at mean light is the average of the behaviour at all phases. This result reinforce the use of the PL relation at mean light in the distance determination context. Readers interested in more details of these statistical methods are referred to the papers cited above.

Despite the results of these studies, universality and linearity of the LL remain to be adopted. The slope and zero point of the LL in optical *VI*-bands have been established with a high degree of accuracy from fundamental-mode Cepheids in the LMC galaxy by Udalski (2000). When the ordinary least-squares (OLS) regression was applied to OGLE-II data of Cepheids, over a range of $\log P$ from 0.4 up to 1.5 for the LMC, and from 0.4 up to 1.7 for the SMC, Udalski realised that the standard deviation of the residuals of the LMC data was almost two times smaller than that of the SMC. Despite that the number of Cepheids in the SMC doubles that of the LMC, the greater dispersion of the SMC PL relation is caused by the spatial distribution of Cepheids in the galaxy bar, whose thickness is placed along the line-of-sight with a typical depth of ~ 0.25 mag (Harris & Zaritsky 2006). This fact, and the manner in which the PL relation of the LMC populates for periods greater than 2.5 days, led to adopt the LMC slope value as a universal quantity (Udalski 2000).

These facts and results we have mentioned above, motivate some questions behind this paper: what is the slope of the

LL from a larger sample of Cepheids? More specifically, what is the slope if the Cepheids in the LMC and SMC are considered as a single sample?

In order to test the universality and linearity hypotheses of the LL and answer the previous questions, this paper is structured as follows: a description of OGLE-II Cepheid data of the Magellanic Clouds is presented in the second section. The extension of the OLS regression, that we develop to calculate the slope of the LL from Cepheids in the Magellanic Clouds simultaneously, is found in the third section. Tests of universality and linearity of the LL, as also our main results and a discussion of them, are presented in the fourth section. Finally, our main conclusions are given in the fifth section.

2 DATA

In a study of the PL relation in optical bands using LMC OGLE-III fundamental-mode Cepheids, Ngeow et al. (2009) found a discrepancy in the slopes computed by them and those reported by other authors, due to the different number of Cepheids used in each study. For this reason, to avoid effects of the sample size, in our study we use OGLE-II data to establish a direct comparison between our slopes and those obtained by Udalski (2000), that remain as the accepted universal values.

The OGLE II photometric data for the Magellanic Clouds were obtained between 1997 and 1999, (Udalski, Kubiak & Szymański 1997; Szymański 2005). High quality *BVI* observations of hundreds of variable stars along the galaxies bars were collected with the 1.3-m Warsaw telescope, at Las Campanas Observatory, Chile (Udalski et al. 1999a). A number of epochs between 120 to 360 was obtained in the *I*-band and between 15 to 40 in the *BV*-bands. A search for periodicity in the photometric time-series of the stars was made, using the analysis of variance algorithm (Schwarzenberg-Czerny 1989). The Cepheid variables were selected based on visual inspection of their light curves and their location on the Colour-Magnitude diagram (Udalski et al. 1999b). To select fundamental-mode Cepheids, an analysis of Fourier coefficients was performed. From the OGLE-II on-line archive we select a number of 745 and 1287 fundamental-mode Cepheids, belonging to the LMC and SMC, respectively. Their dereddened mean magnitudes were computed once the extinction effect was measured (Udalski et al. 1999b, Udalski et al. 1999c). To compute the A_I extinction coefficient, it was determined the *I*-band magnitude of Red Clump stars in many lines-of-sight. Differences of the observed *I*-band magnitude of Red Clump stars were assumed as differences in the A_I extinction coefficient. To compute the colour excess $E(B - V)$ and the extinction coefficient A_V in each line-of-sight, they adopted the reddening law of Schlegel, Finkbeiner & Davis (1998).

3 LINEAR REGRESSIONS

To test the universality and linearity hypotheses of the LL we are interested in obtaining the slopes of the PL relations

in optical bands without adopting the values of the slopes reported by Udalski (2000). With this purpose, we apply two different approaches of linear fits: the OLS regression and the multiple least-squares (MLS) regression, that is an extension of the OLS regression developed by us. The MLS regression is a similar approach to the testimator method described in the introduction section, but with these method we want to find the slope of the PL relation when the mathematical union of data sets of Cepheids from different galaxies is used, under the hypotheses that all data sets share the same slope (universality hypothesis), but each one has a different zero point.

Before presenting the MLS regression, we briefly recall how the OLS regression works, emphasising on some useful results.

3.1 Ordinary Least-Squares Regression

Isobe et al. (1990) stated that the OLS regression should be used to predict the value of one variable from the measurement of another, under the following conditions:

- (i) The nature of the linear regression scatter is not understood, and this scatter is always greater than the errors of the measured variables.
- (ii) It is well established that one variable is the cause and the other is the effect.

As a first approximation, this is the case of the LL: we want to predict the mean magnitudes of Cepheids from their measured periods. To do this, it is necessary to use the OLS regression over a set of N Cepheids, each one of them characterised by two variables: $\log P_k$ and $\langle m \rangle_k$, where the k sub-index takes values between 1 to N . The logarithm of the Cepheid period is $\log P_k$, and in the context of this study, $\langle m \rangle_k$ is the dereddened mean magnitude in a photometric band. The residuals of the measurements of dereddened mean magnitudes, d_k , are defined as the square of the differences between the observed values of these mean magnitudes and the values predicted by the OLS regression, as follows:

$$d_k = (\langle m \rangle_k - \eta \log P_k - \xi)^2. \quad (1)$$

The standard deviation of the residuals is given by:

$$\sigma_{\langle m \rangle} = \sqrt{\frac{S}{(N-2)}}, \quad (2)$$

where S is the sum of the residuals d_k . The expressions of the zero point, ξ , and the slope, η , can be found by minimising d_k . The well know results are:

$$\xi = \frac{1}{N} \left(\sum_{k=1}^N \langle m \rangle_k - \eta \sum_{k=1}^N \log P_k \right), \quad (3)$$

$$\eta = \frac{\sum_{k=1}^N (\log P_k \langle m \rangle_k) - \frac{1}{N} (\sum_{k=1}^N \log P_k) (\sum_{k=1}^N \langle m \rangle_k)}{\sum_{k=1}^N (\log P_k)^2 - \frac{1}{N} (\sum_{k=1}^N \log P_k)^2}. \quad (4)$$

It is possible to express the standard deviations of the slope and zero point in terms of the standard deviation of the residuals (Taylor 1982), as follows:

$$\sigma_\eta = \sigma_{\langle m \rangle} \sqrt{\frac{N}{\Delta}}, \quad (5)$$

$$\sigma_\xi = \sigma_{\langle m \rangle} \sqrt{\frac{\sum_{k=1}^N (\log P_k)^2}{\Delta}}, \quad (6)$$

where Δ is given by the following equation:

$$\Delta = N \sum_{k=1}^N (\log P_k)^2 - \left(\sum_{k=1}^N \log P_k \right)^2 \quad (7)$$

3.2 Multiple Least-Squares Regression

Let G be a set of galaxies, each one having a different number N_l of Cepheids. We want to find an analytical expression for the slope of the PL relation resulting from the mathematical union of Cepheid data of all G galaxies, under the assumption that all data sets share the same slope η . We call this value the common slope. We also want to find the analytical expression for the zero point ξ_l corresponding to each galaxy, under the assumption that each galaxy is at a different distance. For the l th galaxy, the PL relation can be described as:

$$\langle m \rangle_{kl} = \eta \log P_{kl} + \xi_l, \quad k = 1, 2, \dots, N_l \quad l = 1, 2, \dots, G. \quad (8)$$

$\langle m \rangle_{kl}$ is the predicted dereddened mean magnitude of the k th Cepheid belonging to the l th galaxy, from its measured period P_{kl} . In analogy with equation (1), we define the residuals of the measurements of dereddened mean magnitudes for the l th galaxy, as:

$$D_l = \sum_{k=1}^{N_l} (\langle m \rangle_{kl} - \eta \log P_{kl} - \xi_l)^2. \quad (9)$$

We also define the sum of the residuals for all galaxies as:

$$\Upsilon(\eta, \xi_l) = \sum_{l=1}^G D_l. \quad (10)$$

To find the values of the common slope η and the zero points ξ_l , Υ must be minimised with respect to each one of the $1+G$ variables (η, ξ_l):

$$\frac{\partial \Upsilon}{\partial \eta} = 0 \quad , \quad \frac{\partial \Upsilon}{\partial \xi_l} = 0. \quad (11)$$

By solving these equations, the expression for the zero points ξ_l of each galaxy is obtained as:

$$\xi_l = \frac{1}{N_l} \left(\sum_{k=1}^{N_l} \langle m \rangle_{kl} - \eta \sum_{k=1}^{N_l} \log P_{kl} \right), \quad (12)$$

which is a similar expression to the equation (3), given by the OLS regression, but with η being the common slope which is independent of the zero points and that is given by the following equation:

$$\eta = \frac{\sum_{l=1}^G \sum_{k=1}^{N_l} (\log P_{kl} \langle m \rangle_{kl}) - \omega}{\sum_{l=1}^G \sum_{k=1}^{N_l} (\log P_{kl})^2 - \sum_{l=1}^G \frac{1}{N_l} (\sum_{k=1}^{N_l} \log P_{kl})^2}, \quad (13)$$

where

$$\omega = \sum_{l=1}^G \frac{1}{N_l} \left(\sum_{k=1}^{N_l} \log P_{kl} \right) \left(\sum_{k=1}^{N_l} \langle m \rangle_{kl} \right). \quad (14)$$

The equations (3) and (4) can be obtained of the equations (12) and (13) considering a unitary set of galaxies. In the

case of the MLS regression it is possible to obtain an expression for the standard deviation of the residuals generalising the equation (2), taking into account that it is necessary to include the data points of all PL relations. This expression for the standard deviation of the residual magnitudes can be written as:

$$\bar{\sigma}_{\langle m \rangle} = \sqrt{\frac{\Upsilon}{\sum_{l=1}^G (N_l - 2)}}. \quad (15)$$

The standard deviations of the common slope η and the zero points ξ_l can be obtained by generalising the equations (5) and (6):

$$\bar{\sigma}_{\eta} = \bar{\sigma}_{\langle m \rangle} \sqrt{\frac{\sum_{l=1}^G N_l}{\sum_{l=1}^G \Delta_l}}, \quad (16)$$

$$\bar{\sigma}_{\xi} = \bar{\sigma}_{\langle m \rangle} \sqrt{\frac{\sum_{l=1}^G \sum_{k=1}^N (\log P_{kl})^2}{\sum_{l=1}^G \Delta_l}}, \quad (17)$$

where Δ is given by equation (7).

The above expressions are useful in order to test the universality and linearity hypotheses of the PL relation. In order to achieve that, we apply the MLS regression on Cepheids of the LMC and SMC, as a statistically sample of Cepheids. In this case, the l index runs up to 2. We will refer hereafter to the data set formed by the mathematical union of the LMC Cepheids with the SMC Cepheids as the *LMC + SMC* set. At this point, it is important summarise the main characteristics of the linear regression methods exposed. The OLS regression allows to obtain the slope and zero point of the PL relation of a single galaxy. By applying this method to the SMC, an increasing of the dispersion of the PL relation caused by the distribution of the Cepheids along the galaxy bar is noted. In order to obtain simultaneously the slope and zero points of the PL relations using the LMC and SMC Cepheids, and damping the effect of the dispersion caused by the SMC Cepheids, we use the MLS regression over the *LMC + SMC* set.

This method allows us to test the universality of the LL in *VI*-bands making a study different to the traditional ones: we want to establish if the slopes obtained from the *LMC + SMC* set are consistent with the slopes reported by Udalski (2000) for the LMC galaxy, despite of the metallicity differences of the Magellanic Clouds.

3.3 Testing the Multiple Least-Squares Regression

Before applying the MLS regression to the observational data, we perform some tests of the MLS regression over simulated data sets in order to find the common slope and the zero points of each set and verify the validity of the results of this regression method. First, ten sets of PL relations, each one with 10000 data points, are randomly generated. Each data set has a different zero point and the same slope. Then the MLS regression is applied, obtaining the same values of slope and zero points used to generate the PL relations. Then, random noise is added to the generated periods, and the MLS regression is applied again. The slopes and zero points obtained using the MLS regression are the ones expected. In these tests we do not observe trends in the residuals of the mean magnitudes with respect to $\log P$. Finally, we

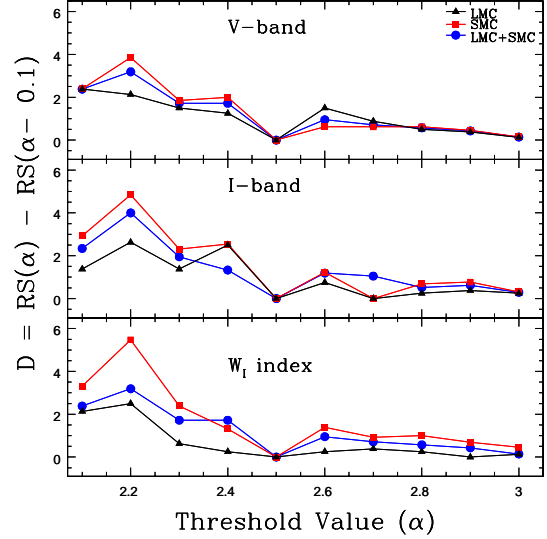


Figure 1. Percentage difference of removed stars vs. threshold value. Results for the *VI*-bands and the W_I index are shown in the upper, middle and bottom panels, respectively. D presents a local minimum in $\alpha = 2.5$. Triangles (black) correspond to the LMC Cepheid data, squares (red) show the SMC Cepheid data and circles (blue) correspond to the *LMC + SMC* set. A colour-version of this figure is available in the on-line journal.

increase the number of sampling points by one order of magnitude. As expected, we observe a corresponding decrease of the standard deviations of slopes and zero points, as well as in the dispersion of the residuals of the mean magnitudes. As the MLS regression works properly with synthetic data, we apply it to the observational data sets of Cepheids in the Magellanic Clouds.

3.4 Removing outliers in the PL Relations

Before to obtain the slopes and zero points of the LL using the OLS and MLS regressions, the outliers must be discarded. To remove them, we find the standard deviation of the residual magnitudes, $\sigma_{\langle m \rangle}$ and $\bar{\sigma}_{\langle m \rangle}$, given by equations (2) and (15). Then, points with a dispersion larger than $\alpha\sigma_{\langle m \rangle}$ and $\alpha\bar{\sigma}_{\langle m \rangle}$, are removed. In order to find the optimum threshold α , we study the behaviour of the percentage difference of removed stars vs. α , selecting α between 2 and 3, with steps of 0.1. Denoting by $RS(\alpha)$ the percentage of removed stars with threshold α , and $RS(\alpha - 0.1)$ the percentage of removed stars with threshold $\alpha - 0.1$, we define the percentage difference as: $D = RS(\alpha) - RS(\alpha - 0.1)$. Fig. 1 shows a plot of D vs. α for the LMC, SMC, and the *LMC + SMC* set, in the *VI*-bands and the W_I index. By examining this figure, it is clear that the local minimum in the studied range lies in $\alpha = 2.5$. For this minimum value there are equal numbers of rejected points between the adjacent threshold values of 2.4 and 2.5, indicating that it is the optimum threshold value. It is important to note that this threshold value is the same for the three bands studied, and is equal to the value reported by Udalski et al. (1999b).

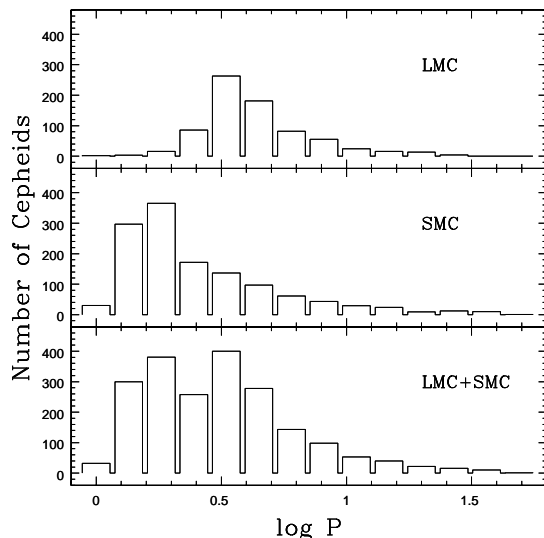


Figure 2. Histograms of $\log P$ for Magellanic Clouds Cepheids observed by the OGLE-II project. The LMC sample presents a sudden decrease of detected Cepheids for $\log P \lesssim 0.4$ (upper panel). For the SMC and the $LMC + SMC$ set, the number of detected Cepheids decreases for $\log P \lesssim 0.2$ (middle and bottom panel).

3.5 Selecting Appropriate Ranges of Periods

In order to obtain slopes and zero points of the PL relations, it is necessary to establish the range of periods that should be selected to apply the linear regressions. Since Cepheids with shortest periods are faintest, they may have been difficult to detect with the 1.3-m Warsaw telescope, used by the OGLE project. Cepheids with longest periods, that are too bright, were over-exposed. Fig. 2 shows a strong decrease in the number of detected Cepheids in the LMC galaxy about $\log P = 0.4$, and in the SMC galaxy and the $LMC + SMC$ set about $\log P = 0.2$. The saturation level of the CCD used by the OGLE-II project, determined the upper limit of detection in $\log P = 1.5$ for the LMC and $\log P = 1.7$ for the SMC (Udalski et al. 1999a).

3.6 Discarding Cepheids with $\log P > 1.5$

We also study Cepheids with periods larger than the measured by the OGLE-II project ($\log P > 1.5$), using photometric data from the third phase of the All Sky Automated Survey (ASAS-3, Pojmański 2012). This data base contains photometry from almost ten years of continuous observations. The ASAS-3 VI magnitudes are near to the Johnson’s standard system, but they were computed without include colour terms in their calibration.¹ The number of Cepheids that we find with periods between 30 and 100 days is 19 and 7 for the LMC and SMC, respectively. Our search for periodicity is made using the analysis of variance algorithm by Schwarzenberg-Czerny (1989).

According to the photometric accuracy and the phase coverage, we try fit each light curve by Fourier series of orders

ranging from 3 to 6. The mean magnitudes are computed following the method described by Gieren et al. (2004). For each one of the ASAS-3 Cepheids, the best fit was obtained using a Fourier series of 4th order. The mean magnitudes obtained from other orders, produce very small effects on the LL. We determine that the changes induced on the zero points and slopes by using these mean magnitudes are of the order of hundredths.

When the ASAS-3 Cepheids are included in the PL relations, they are placed too far from the trend of Cepheids observed by the OGLE-II project. There are two main causes that could explain this behaviour. The first one is the difficulty to correct by extinction, because these Cepheids are placed far from the galaxies bars, where the extinction has been determined with less accuracy. The second one is the large errors (up to several tens of magnitudes) found in the transformation of the instrumental magnitudes to the standard system for some stars (Pojmański & Maciejewski 2005). Based on these causes, we decide not to include the ASAS-3 Cepheids in our study.

Karczmarek et al. (2011) calculated the slope of the PL relation from 65 LMC Cepheids, selected from the ASAS catalogue of variable stars. They found a slope value of $\eta_{ASAS} = -2.366 \pm 0.166 \text{ mag/dex}$ for the V -band. This value differs in more than 10σ from the value reported by Udalski (2000) and from the value of the slope calculated by us with MLS regression (see Table 1). This fact supports our decision to exclude these Cepheids of the sample.

4 RESULTS AND DISCUSSION

In order to test the universality and linearity hypotheses of the LL, we calculate the slopes of the PL relations for the Magellanic Clouds Cepheids through two different approaches. By using the OLS regression it is obtained the slope of the PL relation of each galaxy. By using the MLS regression we obtain the slope value of the LL from Cepheids belonging to the $LMC + SMC$ set.

4.1 Testing the Universality Hypothesis of the Leavitt Law

The universality hypothesis of the LL can be tested by showing that the value of the slope in a photometric band is the same regardless the metallicity of the sample of Cepheids used to derive it. We compute the slope of the Leavitt law for the $LMC + SMC$ set using the MLS regression, assuming the linearity of this law in the VI -bands and in the W_I index, and using the ranges of periods mentioned in subsection 3.5, $0.2 < \log P < 1.7$ for the SMC galaxy and the $LMC + SMC$ set, and $0.4 < \log P < 1.5$ for the LMC galaxy. Table 1 presents the slopes and their standard deviations obtained by us (MLS regression) and by Udalski (2000) (OLS regression). It is worthy of mentioning that the results obtained by Udalski (2000) have been exactly re-obtained in this work. In order to establish a comparison between our results and the reported by Udalski, the slope values η_{MLS} are computed using dereddened VI magnitudes. The first column in Table 1 gives the photometric band in which the LL is studied, the second and third columns give the common slopes and their standard deviations obtained by using

¹ <http://www.astrouw.edu.pl/asas/explanations.html>

Table 1. Slopes obtained testing the universality hypothesis. Columns 2 and 4 give the slope values obtained by us and by Udalski (2000), respectively. A detailed explanation of each column is given in the text.

Band	η_{MLS}	$\bar{\sigma}_\eta$	η_{OLS}	σ_η	N_{LMC}	N_{SMC}
V	-2.806	0.028	-2.775	0.031	700	882
I	-3.024	0.020	-2.977	0.021	705	857
W_I	-3.364	0.012	-3.300	0.011	729	821

Table 2. Asymptotes μ obtained testing the linearity hypothesis. A complete explanation of each column is given in the text.

Band	LMC		SMC		LMC+SMC	
	μ	$\Delta\mu$	μ	$\Delta\mu$	μ	$\Delta\mu$
V	-2.726	2.751	-2.759	0.025	-2.767	0.062
I	-2.909	2.130	-3.008	0.029	-2.993	0.040
W_I	-3.311	0.005	-3.376	0.013	-3.358	0.009

the equations (13) and (16). Columns fourth and fifth give the slopes and their standard deviations for the LMC obtained by Udalski (2000), applying the equations (4) and (5). Columns sixth and seventh give the total number of Cepheids in the LMC and SMC galaxies that we use with the MLS regression, after rejecting outliers. Our computed common slope η_{MLS} of the LL is consistent at a level of 1.0σ and 2.2σ in V- and I-bands, respectively, with those reported by Udalski (2000). This implies that the slope of the PL relation in the VI-bands is the same independently of the metallicity of the Magellanic Clouds. On the other hand, the slope η_{MLS} in the W_I index is not consistent with that of Udalski (2000). These facts indicate that the LL is universal in the VI-bands but not in the W_I index. The slope obtained in I-band is not as accurate as the reported by Udalski, probably due to blending effects reinforced by the bar geometry of the SMC galaxy. If Cepheids are blended with red stars, this may change their mean I magnitudes increasing the dispersion of the PL relation.

4.2 Testing the Linearity Hypothesis of the Leavitt Law

The linearity hypothesis of the LL can be tested by showing that the value of the slope in a photometric band is the same regardless of the range of periods selected to derive it. In order to compute the slope, we select the range of periods between a lower limit and an upper limit. The lower limit is useful to avoid the effect of Malmquist bias on the slope of the LL, that makes the slope value too shallow. This effect is present when the faintest Cepheids in a galaxy are very close to the cutoff magnitude of the photometry (Pietrzyński et al. 2010). According to the discussion in subsection 3.5, we take the lower limit for the range of periods at $\log P = 0.4$ for the LMC, and $\log P = 0.2$ for the SMC and the LMC + SMC set. We select the upper period limit of the Cepheids ranging from $\log P = 0.5$ up to $\log P = 1.5$, with steps of 0.1, for the LMC, SMC and the LMC + SMC set.

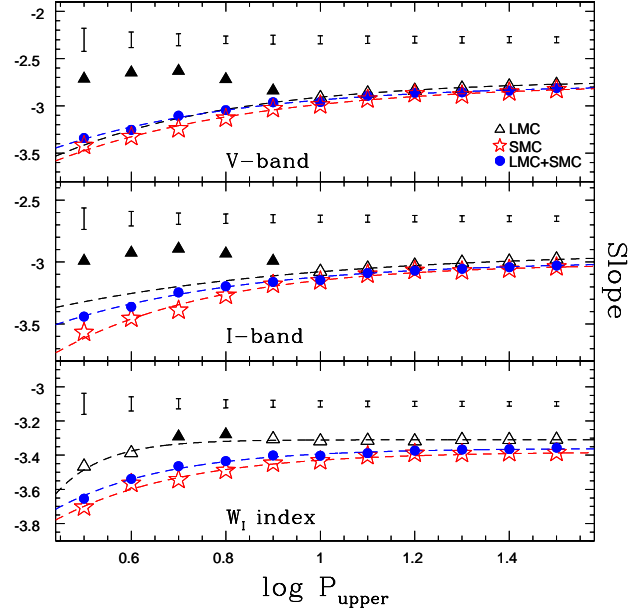


Figure 3. Slope vs. logarithm of the upper limit for range of periods. Triangles and stars represent the slopes obtained through the OLS regression for the LMC and SMC, respectively. Filled circles show the common slope obtained with the MLS regression, for the LMC + SMC set. Filled triangles are identifying the LMC outliers data, see text for explanation. Dashed lines (black for the LMC, red for the SMC and blue for the LMC + SMC set) are the exponential fits given by equation (18). Upper, middle and bottom panels show the behaviour of the slope of the LL for the VI-bands and for the W_I index, respectively. In each panel are shown the slope errors obtained by using the MLS regression over the LMC + SMC set. Large error bars correspond to slopes computed with few data between the range of periods. The slope errors corresponding to the LMC and SMC, are similar to the errors of the LMC + SMC set. They are omitted to appreciate clearly the figure. A colour-version of this figure is available in the on-line journal.

Fig. 3 shows the slope values for the PL relations as a function of the logarithm of the upper limit of period. Each one of these slope values for LMC and SMC samples was obtained by using OLS regression, and the slope values for the LMC + SMC set were obtained using the MLS regression. The behaviour of the obtained slopes is well fitted by an exponential function (dashed lines in Fig. 3):

$$\eta = Ae^{-\beta \log P} + \mu, \quad (18)$$

where the A , β , and μ parameters are obtained using a non-linear fitting with the IRAF task *nfit1d*. The A parameter gives information about the stretching or squeezing of the function and the reflection about the horizontal axis. The β parameter gives the reflection about the vertical axis. The asymptote μ can be interpreted as follows: if the Cepheid population of a galaxy is observed, including all Cepheids with periods up to 100 days, and their mean magnitudes and periods are used in order to obtain the slope of the PL relation, this slope value approaches to μ .

Inspecting the Fig. 3, it is clear that the LMC LL presents a break around 10 days, in VI-bands. This result is in agree-

ment with those reported by Tammann, Sandage & Reindl (2003), Kanbur & Ngeow (2004), Kanbur et al. (2007) and Koen, Kanbur & Ngeow (2007).

The LMC slopes in VI -bands behave anomalously for short-periods ($\log P < 1.0$, black triangles in Fig. 3), in the sense that they move away from the exponential function which adjusts the data trend for long-periods ($\log P > 1.0$). For this reason short-period points for the LMC are considered to be outliers to fit the data by the equation (18). On the other hand, the obtained slopes by the OLS regression for the LMC are shallower than the slopes of the SMC for short-periods. The causes of this behaviour are still under investigation by us.

The behaviour of the slopes shown in Fig. 3, provides clear evidence of the non-linearity of the LL in the VI -bands and in the W_I index for the Cepheids belonging to the Magellanic Clouds. This result about the non-linearity of the LL is in concordance with the corresponding non-linearity of the PC relation, reported by Kanbur & Ngeow (2004) and Koen, Kanbur & Ngeow (2007) for the LMC Cepheids. Besides, due to that the PC and the PL relations are projections of the fundamental PLC relation (Madore & Freedman 1991), our result of non-universality of the PL relation suggests that the PC relation should be non-universal in the W_I index. In Table 2 we present the values of the asymptote μ and their errors obtained by using a non-linear fit with the IRAF task *nfit1d*. The errors of the asymptote μ , $\Delta\mu$, are computed taking into account the slope errors. The first column shows the photometric band where the LL is studied. The second and third columns give for the LMC the values of μ and $\Delta\mu$. The large $\Delta\mu$ values for the LMC reflect that, in that case, the μ values are obtained using about one half of the points that are used in other fits. The following pairs of columns in Table 2 give the same information that the first two columns, but for the SMC and the $LMC + SMC$ set.

Despite the non-linearity of the PL relation and its break around 10 days (for the LMC), its use to determine distance is not significantly impacted by these effects due to the following reason:

The calculation of distance modulus of a galaxy, relative to the LMC, is performed from the difference between zero points of the PL relations of the galaxy and the LMC. These zero points are function of the slope of the PL relation as shown the equations (3) and (12).

The slope of the PL relation is influenced strongly by long-period Cepheids. In a galaxy these Cepheids are less numerous than short-period Cepheids and are distributed in a wide range of periods. Therefore, a small number of them can produce a significant change in the slope value more than a large number of short-period Cepheids.

The effects of non-linearity of the PL relation can be estimated computing the deviation of the slope from the value of μ . Inspecting Fig. 3, it is seen that the bigger is the period range used to derive the slope of the PL relation, the smaller is the effect of non-linearity. For example, in V -band the deviation of the slope obtained with MLS regression from the value of μ , in the period range from 2.5 to 10 days, is 3σ , while for a period range from 2.5 to 25 days, is 1.1σ .

The slopes of the PL relation calculated by Udalski (2000) were obtained with Cepheids up to 32 days (LMC) and 50 days (SMC), for which the effect of non-linearity obtained

in the present work is in the order of one sigma in V -band. Based on these facts our results reinforce the use of the LL to determine distances, independently of the non-linearity and the break of the PL relation.

It can be seen from Table 2 that the values of the asymptotes μ for the $LMC + SMC$ set, in VI -bands, agree in less than 1.0σ with the values for the slopes of the PL relations found by Udalski (2000) (see Table 1, fourth and fifth columns). The μ values obtained for the LMC and SMC are consistent with the values of η published by Udalski (2000), up to 1.6σ for the V -band, and 3.2σ for the I -band. Also, it can be seen from Table 2 a statistically significant difference between the μ values of the LMC and SMC in the W_I index, and also between the μ value of each galaxy with the η value published by Udalski (2000). These results suggest that the slope μ can be considered as universal in VI -bands but not in the W_I index. In a forthcoming paper we will be applying the OLS and MLS regressions in galaxies other than the Magellanic Clouds, in order to test the universality and linearity of the LL in these metallicity environments.

5 CONCLUSIONS

In this work we test the universality and linearity hypotheses of the LL, using the sample of Cepheids belonging to the Magellanic Clouds, observed in VI -bands by the OGLE-II project. In order to develop these tests, we compute the slope values of the LL using two different approaches. One of them makes a mathematical union of the Cepheid data of the LMC and SMC galaxies to find a common slope applying the MLS regression. The other one obtains the slope of the PL relation applying the OLS regression on a single galaxy: the LMC or the SMC.

The test of the universality hypothesis lead us to obtain a common slope for the $LMC + SMC$ set using the MLS regression. Our values are consistent with the reported by Udalski (2000) for the LMC at a level of 1.0σ and 2.2σ in V - and I -bands, respectively, and inconsistent in the W_I index. Our results suggest a strong dependence on metallicity of the slope for the W_I index, and a weak dependence on metallicity in the optical VI -bands, in agreement with the results reported by Storm et al. (2011), reaffirming the universality of the LL in the VI -bands and showing again its non-universality in the W_I index.

The test of the linearity hypothesis lead us to find that the values of the slopes of the LL behave exponentially as the range of periods increases, giving clear evidence of non-linearity. In particular, we find that the LMC LL presents a break around 10 days, according to the results reported by Tammann, Sandage & Reindl (2003), Kanbur & Ngeow (2004), Kanbur et al. (2007) and Koen, Kanbur & Ngeow (2007). A clear explanation of the behaviour of the LMC PL relation, in the range of short-periods is unknown until now. Our result about the non-linearity of the LL is in concordance with the corresponding non-linearity of the LMC PC relation, reported by Kanbur & Ngeow (2004) and Koen, Kanbur & Ngeow (2007), and suggests that in the W_I index the PC relation should be non-universal.

Despite of the non-linearity of the LL, we find that the asymptote μ has values in VI -bands that are consistent with the slopes reported by Udalski (2000) for the LMC, but in-

consistent in the W_I index; therefore, in VI -bands, μ can be considered as universal and the LMC slopes given by Udalski (2000) remain appropriate to measure extragalactic distances. However, it is necessary to take into account that the universality of the LL has been tested only in a few galaxies; therefore it will be necessary to study the PL relation in many galaxies other than the Magellanic Clouds, in order to test the universality of the LL in different metallicity environments.

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